1. UTMOST Project Description
Undergraduate Teaching in Mathematics with Open Software and Textbooks

Software for mathematical explorations, including computer algebra systems (CAS), have held great promise for education since their first appearance in the 1960s (Reduce, Macsyma [51, 65]) and the introduction of Maple and Mathematica in the 1980s [49, 89]. Currently, CAS are widely found in the undergraduate classroom and a considerable amount of mathematics education research has focused on the use of CAS in learning undergraduate mathematics. Studies have documented the improvement in students' understanding [33, 34, 43, 57]. For example, the use of technology allows students to avoid tedious or unreasonable computations, avoid long tables of integrals or Laplace transforms, visualize complicated 2-D plots and 3-D mathematical objects, and easily deal with unwieldy matrices in linear algebra. Another example is that technology has allowed a qualitative approach to teaching differential equations. Most differential equations cannot be solved explicitly, but numerical and graphical software has enabled teachers to incorporate a qualitative approach in the courses that they teach [7]. The ability to make large computations, quickly and without errors, with graphical output, can greatly aid students' understanding of difficult ideas in mathematics and provides them with an incredible capacity for experimentation and conjecture.

Computer algebra systems are now common in the undergraduate classroom, yet we rarely see a seamless integration with the curriculum. To accommodate competing commercial systems that divide the market, textbooks typically offer supplements for several different CAS, or they are "technology-enhanced" with generic sidebars. Licensing restrictions for campus use, the expense of personal copies, and underpowered hardware often mean that students can only work with commercial software in campus labs. Some institutions are unable to afford the cost of building such labs for their students.

There have been efforts at seamless integration of technology and curriculum (e.g., calculus [81]). An example of such an experiment is the Calculus & Mathematica project, which generated much excitement in the early 1990s and had demonstrable success in helping diverse students learn more effectively [2, 58]. However, today its use seems limited to the two institutions where it originated [55, 86]. Likewise, the use of interactive Java applets to support teaching mathematics (such as [3, 10, 35]) does not seem to have been widely adopted. The undergraduate curriculum has not seen the broad transformative effect of these powerful tools for increasing the learning and understanding of mathematics. We propose an approach using open source software which we believe will allow for the wide adoption of approaches that have proven to be successful.

1.1. **An open approach.**

Sage [5, 75] is free open source software designed to be an alternative to Magma, Maple, Mathematica, and Matlab. Coinciding with the development of Sage, there is a general movement to freely-available open textbooks that includes many quality texts in mathematics ([12, 56], Section 5.1.3). Our proposition is that freely-available open software, open textbooks, and other open curricular materials can allow teachers everywhere to transform the undergraduate mathematics curriculum by tightly and seamlessly integrating mathematics software with more traditional curricular materials. We will test this hypothesis by integrating Sage into existing open textbooks and other curricular materials, placing the full computational power of Sage *directly into* a student's text, usable any time from anywhere via a web browser. For the institution and the instructor, the cost and time-consuming licensing inconveniences of commercial software are removed by open software. Sage can also be used remotely from a standard web browser, eliminating the need for a dedicated local computer lab. Furthermore, the essential nature of open software means that curricular decisions and needs can drive the development of the software, with the classroom teacher actively advising (or actually doing) the software development. Likewise, open curricular materials can be adapted to course needs and teacher preferences, and can be distributed with the software freely in an integrated package. Leveraging these inherent advantages of an open approach to software and curricular materials, the promise of mathematics software in education can be fully realized by faculty and students. Our work will create and disseminate a model for this integration, addressing both the pedagogical and technical aspects, so that other faculty authors can realize the advantages in their own curricular materials and courses.

More specifically, UTMOST will build and test a model for easily integrating open mathematics software and open educational materials into the mathematics curriculum and classroom as follows.

- We will create a system that makes it easy for authors to convert open textbooks and other curricular materials to Sage worksheets, interspersing runnable interactive demonstrations and exercises and live Sage code with publication-quality typeset mathematics.
- We will convert existing mature open textbooks to this format and create new curricular materials targeting this format, as demonstrations and tests of both the technical and pedagogical aspects of this new approach.
- We will partner with ten diverse institutions to test these materials in a wide variety of courses, and provide support for their use and assistance for the creation of new materials.
- We will evaluate the effectiveness of our model for making it easy to adopt open mathematics software and textbooks and making it easy to create integrated open curricular materials, and we will measure the resulting impact on teaching practices and the learning of mathematics with the expert assistance of professional evaluators.

In addition to workshops and other presentations, materials created or enhanced by UTMOST will be widely distributed with open licenses and made available through the global Sage website (`www.sagemath.org` [75]). As appropriate, they will also be available in Sage itself or through a new open textbook initiative at the American Institute of Mathematics [1] as part of this project.

2. Trends in Scientific Communication

The cost of academic research journals, especially in science, combined with dissemination restrictions imposed by copyright law and new dissemination possibilities afforded by technology, are collectively referred to by librarians as the "serials crisis" [9]. Faculty have come a long way in their efforts to return scientific publication to a free exchange of ideas. Electronic journals and public repositories now publish articles with licenses that explicitly allow for sharing new results easily via the Internet without distribution fees. Government initiatives, such as the Public Access Policy of the National Institutes of Health [54] and the Policy Forum on Public Access of the White House Office of Science and Technology [88], are working to accelerate this trend.

Following on the sea change in research publication, the next wave is open textbooks. Faculty are all too familiar with the problems that plague commercial textbook publication, such as high prices, edition churn, and orphaned works. Open licenses are now being used to assert control of these critical resources for education, with faculty in mathematics, computer science, and business as the leaders [22, 79, 85]. Government is poised to accelerate this open textbook trend at all levels, with Senator Durbin proposing legislation directing the Department of Education to award grants for the creation of open textbooks by faculty [19], Washington State's initiative to provide open textbooks for the eighty highest-enrollment courses in its community college system [87], and California's initiative to create free digital textbooks for its high schools [71].

With the emergence of viable and comprehensive open source software for mathematics, there is now a spectacular opportunity for mathematics teachers to use, extend, and create this important software for mathematics education and shape it to reach its full potential in the service of educating students. UTMOST will create an easy path for all faculty to make the initial transition to open software, open textbooks, and open curricular materials in their courses. This path will make it easy for diverse schools and faculty to employ mathematics software to transform the classroom into an interactive lab which takes the study of mathematics to a new level.

2.1. **Imagine this!** A student is learning about row-reducing matrices in a beginning linear algebra course. The electronic version of their textbook is an online Sage worksheet they can view from anywhere. Mathematical equations, with publication-quality typesetting, describe the procedure. An interactive demonstration, embedded in the worksheet where an example normally would be, allows the student to step through row-reducing a matrix that was generated on-the-fly (see the second screenshot in the supplementary documentation). When ready to guide the procedure themselves, the student may choose the row operation to apply at each step of the reduction. The correct notation for each operation is displayed and the operation itself is highlighted with color-coded entries in the displayed matrix. Another example shows the student how to use a built-in Sage command to row-reduce a matrix. With a single click, the student creates an empty code cell under the example to experiment with the command.

In class, the instructor does one simple example on the board. Then the instructor opens a Sage worksheet, displaying the same demonstration that is in every student's electronic textbook. Students guide the choice of row operations at each step for several examples of increasing complexity. At the end of class, the instructor clicks a button to publish the work from that day's class session to the class Sage server. After class, students view the worksheet as they review their notes. They can also, with a click, copy the worksheet into their notes, as well as annotate or make changes.

The book's exercises include an interactive problem generator that creates matrices of student-specified sizes and complexity for the student to row-reduce. There is a "Solution" button which generates a step-by-step solution. An advanced exercise guides a student through investigating numerical issues that arise in row reduction. By changing one parameter within a command, the matrices track numerical error bounds via interval arithmetic, and the student discovers that numerical errors can be a significant problem for some matrices.

A group exercise asks students to collaboratively write and test a short function to implement row-reduction using row operations. The student clicks a button to open a new Sage worksheet and types in a few lines using Sage matrices and commands. The student evaluates the code with a single click. Satisfied with their work, the student clicks another button, publishing the worksheet to the rest of the group for testing. Later that evening, the student checks the worksheet and sees that other students have tested the function, found an error, and repaired it.

The next week, the instructor logs into the online Sage server with a web browser and looks over all of the published worksheets for the group projects. The instructor annotates each worksheet online, using an embedded word processor that supports mathematical typesetting [14, 84].

The instructor writes a quiz on row-reduction techniques. Inside the LaTeX file for the quiz, the instructor includes a Sage command to create a simple matrix in reduced-row echelon form, which will be the *answer* to a quiz problem. The instructor then uses Sage commands inside the LaTeX file to do several simple row operations to transform the answer matrix to the matrix the students will row-reduce. The embedded Sage commands are executed automatically when the quiz is formatted with LaTeX [18], and the resulting question matrix is automatically incorporated in the printed quiz, while the answer matrix and a detailed solution are automatically incorporated in the answer key.

3. Why Sage?

Sage is a natural choice for software to realize the benefits of an open approach to the undergraduate mathematics curriculum. Sage is a comprehensive program with an open development process, a modular design philosophy, an easy-to-use interface utilizing standard web browsers, tight integration with LaTeX, and an industry-standard programming language. With an open license, the ability to run on a remote server, and a platform-independent interface, Sage removes substantial financial and logistical barriers to classroom use of mathematics software. In this section, we describe the many features of Sage that make it a good choice for integrating mathematics software with open textbooks and other curricular materials.

3.1. **A comprehensive program.** Sage's modular design allows symbolic, exact, and numerical approaches to mathematics to coexist equally. Mathematical objects, such as functions, differential equations, rings, fields, modules, and vector spaces are "objects" that look and behave as their abstract mathematical definitions intend. Sage incorporates many different computational strategies and so does not rely solely on a single computational strategy, such as pattern-matching, which favors symbolic computation, or floating-point numbers, which favors numerical work.

Institutional, departmental, and classroom decisions about the use of a CAS require a commitment to a particular vendor's vision for the curriculum. With a modular design that allows for packages specializing in different approaches, Sage can seamlessly support many approaches to exploring mathematical problems. Additions and extensions to Sage, driven by real curricular needs, can be easily and quickly incorporated. As individuals and institutions extend the software, the entire mathematics community benefits from a comprehensive program that students and faculty can employ in courses ranging from pre-calculus and introductory statistics through advanced courses like abstract algebra and number theory, as well as in graduate courses and professional careers.

3.2. **Curriculum and open development.** The closed development process for commercial software creates a high barrier for teachers and students to alter or extend the software to meet their curricular needs. For example, while external extensions, such as libraries or packages, can extend proprietary systems, these must be purchased, distributed, and installed by every end-user. However, an open development process allows students and faculty to shape the core technology. Software developed openly and collaboratively, such as Sage, allows the teaching and learning of mathematics to drive the technology, rather than the technology driving the teaching and learning. A concrete example of this important principle is given in Section 4.

3.3. **Building the car, not reinventing the wheel.** Sage unifies over one hundred mature, best-of-breed, open source packages. These packages range from focused libraries that excel at specific types of computations (e.g., Fast Library for Number Theory for integer polynomial arithmetic [23], the Integer Matrix Library for solutions to linear systems over the rationals [36], or M4RI for exact computations with binary matrices [47]) to complete applications or general libraries for broad areas of mathematics (e.g., R for statistics [64], GAP for group theory [25], and SciPy for numeric scientific computations [72]). Sage ties these packages into a single open source system with a consistent interface, making it easy for a teacher or student to smoothly explore vast areas of mathematics seamlessly, using the best tools for each computation. As a student moves from course to course, the Sage notebook interface and commands remain consistent, even if the particular computations may be performed by an entirely different package.

3.4. **Technology independent.** Sage's notebook interface allows a student to communicate with a Sage server through a standard web browser—interactively running commands, viewing textual and graphic output, and annotating their computations with an online word processor that supports mathematical typesetting. The notebook interface removes many of the logistical barriers to using software in the study of mathematics. A student can employ the full power of Sage using a remote Sage server with only a web browser and a minimal network connection on a desktop computer, a small laptop or netbook, or even a cell phone [69]! This flexibility means that instead of an institution providing an expensive lab of machines with commercial software that can only be used on the dedicated computers, any computer can harness the full power of Sage.

The Sage notebook interface relies heavily on industry-standard JavaScript and acts as a web application similar to Google Docs, Gmail, or wiki software. Students have designed and implemented much of the current notebook. The first screenshot in the supplementary documentation shows an example Sage worksheet in the current Sage notebook.

Every copy of Sage includes the Sage server software. To establish a local server, colleges can download and use the ready-made Sage virtual computer image on a computer that supports their needs. There are also publicly-available servers that can be used to support students and classes. The flagship public Sage server is `www.sagenb.org`, a $100,000 rack of servers located at the University of

Washington that was fully funded by an NSF SCREMS grant (DMS-0821725) and is currently home to over 21,000 accounts. Because it is publicly accessible, students from the poorest universities all over the world are running computations on the same version of Sage, on the same hardware cluster, with leading researchers in computational number theory. Constructed in January 2009, this powerful server should be a viable resource for many years to come. Publicly-accessible notebook servers continue to appear throughout the world (e.g., in Korea [68] and Hungary [67]) and many more run behind campus firewalls for dedicated use [70]. Since every copy of Sage includes the server software, when there is no network access, Sage can be installed directly on a computer and used as a private server with exactly the same online notebook interface as the remote Sage servers.

The flexibility of freely-available online access is a key consequence of the open nature of Sage and its importance cannot be overemphasized in education. As student populations become increasingly mobile, having student-driven technology that can be used freely from anywhere, especially from underpowered mobile devices, supports natural trends in student life.

3.5. **Communicating mathematics.** TEX, with its add-on package LATEX, is another open source success story, and is the typesetting language of choice for mathematicians and other technical disciplines. Every mathematical object in Sage can be typeset in LATEX automatically. The Sage notebook uses jsMath [14] to typeset mathematics beautifully in a Sage worksheet (within a standard web browser). Furthermore, the Sage notebook interface allows a user to insert new text and automatically typesets mathematics. This is another example of how open standards and open software combine to make powerful tools, and is especially relevant for our plan to convert LATEX documents into Sage worksheets.

In addition to using LATEX in the notebook interface, Sage can be used from within any LATEX file to perform computations and create figures. Dan Drake, a Visiting Professor at Korea Advanced Institute of Science and Technology, authored SageTEX [18], a LATEX package which allows Sage code in the file to easily be run as part of the process of formatting a LATEX document. Output (as typeset mathematics or images) is then automatically embedded in the resulting document.

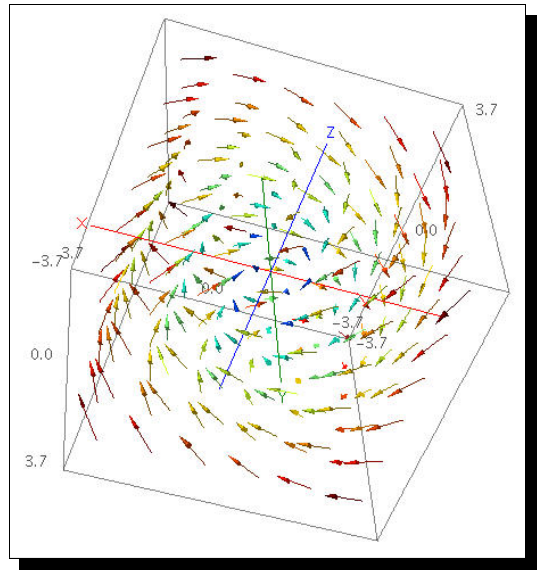
3.6. **A standard programming language.** The many components of Sage are held together with a significant library of new code, written in an industry-standard programming language Python [61]. Through the power of Python, new open source packages written in a variety of programming languages are easily added to Sage, bringing users new functionality or improving existing functionality. More and more packages are appearing for mathematics and science written in Python, often with open licenses [63, 72].

Students may use Sage through point-and-click interactive demonstrations written by others, or they may execute a sequence of single-line commands in the notebook. However, for more involved computations, they can use Python, since it also serves as the user language in Sage. This is in contrast to other comprehensive programs for mathematics that have chosen to create and develop their own programming languages, which are of no use outside of the CAS. A user with knowledge of Python is ready to be productive in Sage immediately, while a student using Sage that is new to programming receives a basic familiarity and education in Python—a skill that is readily transferable to mathematical and non-mathematical applications in a wide variety of fields, including art, business, science, engineering, and many other disciplines [62].

4. An Example of Teachers Driving Open Development

In this section, we provide a concrete example of how teachers and classroom needs drive the development of Sage to directly create better and more comprehensive tools for use in the teaching and learning of mathematics.

Jason Grout, then a postdoctoral associate at Iowa State University, was asked by a meteorology student if Sage had a 3-D vector field plotting function. Grout quickly wrote a simple one. Open development communities encourage sharing even experimental code, so he posted his code online in Sage's publicly-accessible online database of enhancements and bugs. Robert Bradshaw, a graduate student at University of Washington who had written much of the 3-D graphing code in Sage, suggested a small change that greatly increased the efficiency of the function. Several months later, another person authored a very similar function which had slightly improved options and published their code on the public Sage notebook server. In Fall 2008, Grout needed the 3-D vector field plotting function for his multivariable calculus class. He made a few suggested changes to the function and posted it for his class to use.



$\vec{f}(x, y, z) = y\vec{\imath} + z\vec{\jmath} + x\vec{k}$ plotted with Sage's `plot_vector_field3d` command

In the next few months, improvements were made to the internals of Sage 3-D graphics. Many of these improvements were made by William Cauchois, a University of Washington freshman supported for a summer by an NSF VIGRE grant, and others were made by Jason with an eye towards simplifying the vector field plotting function code. Jason then taught multivariable calculus again. He incorporated the best ideas from the various sources and posted a documented, efficient version of the function, this time asking the community for a formal review (a prerequisite for his code to be added to the core Sage library). Marshall Hampton, an Assistant Professor at University of Minnesota, Duluth, reviewed the function immediately, with the comment "Very nice, positive review. This is great timing since I am about to teach vector fields in a week or two." The function was incorporated into the next release of Sage, two weeks later, ready for use by the entire Sage community.

Not only do the original students benefit from the invested effort, but the collaborative effort of at least five teachers and students directly led to a function that will serve all who use Sage. This example illustrates many strengths of the open development process, where teachers and curriculum drive the technology. Such activities are generally not possible with a commercial CAS.

5. Implementation

Our goal is to widely disseminate successful approaches for using technology in the teaching and learning of mathematics. By using open curricular materials, integrated with powerful and comprehensive open software, we will realize a transformative effect. UTMOST will revolve around three coordinated efforts: creating Sage-enhanced curricular materials; working with and providing classroom support for teachers using these materials; and providing the Sage infrastructure to support these materials. Specifically, UTMOST can be divided into five main activities.

(1) We will create a system to convert open mathematics textbooks to Sage worksheets that can incorporate live Sage code, as well as interactive demonstrations and exercises powered by Sage. Our work to make these textbooks Sage-enhanced will provide a model for other authors to create Sage-enhanced textbooks. The first courses we will address are linear algebra, abstract algebra, and number theory. See Section 5.1 for details.

(2) We will further implement our model for authoring Sage-enhanced curricular materials by creating modules for standard undergraduate courses, especially courses not covered by the textbooks we are converting. These materials will include subject-specific guides to introduce faculty and students to Sage in the context of a particular topic. These materials will be created by members of the UTMOST team and by faculty at our ten test sites. Topics under consideration include single variable calculus, multivariable calculus, differential equations, and complex variables. See Section 5.2.

(3) We will identify and work closely with teachers at ten different institutions to test, refine, improve, and extend the work described above. We will select a broad range of institutions, including some with diverse student populations. Dedicated Sage workshops ("Sage Days") will provide opportunities for the UTMOST team and representatives from the test sites to collaborate. These ten sites will provide opportunities for our evaluation team to measure the success of our efforts and the impact on teaching and learning. See Section 5.3 for implementation details and Section 6 for evaluation details.
(4) We will disseminate our materials and the results of UTMOST through Sage Days workshops, and we will apply for MAA PREP workshops and MAA minicourses (national and regional). We will use these opportunities to train faculty outside of the ten test sites in the use of these materials for their courses, and in the process for creating their own new materials. Our materials will all carry open licenses and be available through the global Sage website. As appropriate, they will also be available in Sage itself or through a new open textbook initiative created at the American Institute of Mathematics as part of this project. See Section 7.
(5) We will improve the Sage library and its surrounding infrastructure (e.g., server design, notebook usability, and collaboration tools) where the improvements have a direct and obvious benefit for undergraduate education. See Section 8.

5.1. **Sage-enhanced open textbooks.** Our first main activity is the design and creation of Sage-enhanced textbooks.

5.1.1. *Infrastructure for textbooks.* UTMOST proposes creating and fully testing a system for integrating Sage and textbooks. Pilot projects have already demonstrated the feasibility of this process for short documents, and technical improvements have been identified which will make integrating Sage and book-length documents easy for authors. The primary tool is the existing `tex4ht` translator (NSF IIS-0312487) [82], which converts LaTeX into the jsMath format adopted by the Sage notebook for rendering publication-quality mathematics. The end result will be a folder of Sage worksheets, each a mixture of text, typeset mathematics, Sage input cells, and Sage interactive demonstrations and exercises. Since LaTeX is the accepted standard for authoring mathematics, we expect this process to be applicable to a broad range of current and future mathematics textbooks.

5.1.2. *Targeted textbooks.* Listed below are our initial candidates for conversion to Sage-enhanced textbooks that bring the full power of mathematics software to the student *directly within their book.* In addition to converting existing content to Sage worksheets, we plan to add interactive demonstrations and exercises, live Sage code, and guidance on the Sage library. There is significant care and thought required to integrate these uses of mathematics software properly. Our work will provide a tested model for other authors when writing new textbooks. We expect to have the three texts below available for use in the Fall 2011 term at our test sites. Each of these is authored by a project member and so provides an excellent testbed for the enhancement process.

- Beezer's *A First Course in Linear Algebra* [4]. This is a mature textbook designed from the start to be open source and thus has a highly modular design. The first version of this book was released in December 2006 and has been used as the primary text in courses at 13 different universities. The author has taught from various versions of the book ten times. It is an ideal choice as a full-scale test of the technical conversion process, the integration of Sage code, and classroom use.
- Judson's *Abstract Algebra: Theory and Applications* [37]. This book was originally published by PWS-Kent in the early 1990s, but the author has received the copyright back and released it under an open source license. Beezer has taught from this text the past two years and created supplementary material utilizing Sage and contributed code to the Sage library to

support the book. First released as open source in Spring 2009, eight universities adopted the book for the Fall 2009 term.

- Stein's *Elementary Number Theory* [74]. This text has an extensive collection of Sage examples. Additionally, the author has taught from the book four times. It is currently being published by Springer-Verlag, but Stein has the rights to make it freely available starting May 2010, so will be able to create a Sage-enhanced version for free distribution. While it will not carry a true open license allowing modifications by others, it will be a good test of the technical aspects of the conversion process and will be made freely available.

5.1.3. *Open mathematics textbooks.* The catalog of open mathematics textbooks is growing rapidly and will likely have many more entries by Summer 2012 when we will select more textbooks for conversion. There is a wide range of books available now that could be converted to cover parts of the standard undergraduate curriculum, including mathematics for liberal arts [46], trigonometry [15], college algebra [76], single and multivariable calculus [16, 17, 26, 29, 50, 78], linear algebra [4, 32], differential equations [42], numerical analysis [59], number theory [52, 74], cryptography [38], logic [6, 48], probability [27], combinatorics [8], real analysis [20, 21, 41], and abstract algebra [37]. Almost without exception, these textbooks are licensed with a GNU Free Documentation License (GFDL) or a Creative Commons License (CC) that allows anyone to distribute modified versions, typically only requiring attribution of the original author's work and the use of the same license. Authors come from a wide range of institutions, representing community colleges [17, 46, 76], liberal arts colleges [4, 8, 29], and research institutions [41, 74, 78].

5.2. **Sage-enhanced curricular materials.** In addition to enhancing selected open textbooks, we will also enhance and develop smaller-scale curricular materials for other undergraduate courses.

5.2.1. *Subject-specific guides.* For selected courses where we do not provide a Sage-enhanced textbook, or no suitable open book currently exists, we will create subject-specific tutorials. These short guides will explain how to use Sage to investigate questions in a standard course, following a typical order for such a course. Existing examples of these smaller-scale materials include John Perry's course notes for a Mathematical Computing course [60], drafts of multivariate calculus worksheets by Jason Grout and Ben Woodruff [28], and Stein and Mazur's Riemann Hypothesis book [52], which targets curious undergraduates. These will be designed from the start for conversion to Sage worksheets, providing further small-scale tests of our system for conversions. Guides will be authored by UTMOST project members, and faculty at our test sites.

5.2.2. *Interactive demonstrations and exercises.* Sage has a powerful, but extremely simple-to-use framework for creating web-based interactive demonstrations. Input boxes, sliders, selectors, and other controls are simple to create. Output can use HTML, tables, typeset mathematics, and 2-D or 3-D graphics. Computations have the full Sage library at their disposal. The second screenshot in the supplementary documentation contains an example of such an interactive demonstration. Stein created the first implementation and continues to improve it with significant contributions from Grout and many other Sage developers. Current work by Mitesh Patel, supported by an NSF FRG grant (DMS-0757627), will allow embedding these interactive demonstrations into standard web pages, usable without having to log into a Sage server.

For selected courses, we will create comprehensive collections of demonstrations for inclusion in our Sage-enhanced textbooks and into Sage itself. We will also create an easily-searchable repository of high quality, reviewed Sage interacts, which will be included in every copy of Sage.

5.3. **Sage into the classroom.** The heart of UTMOST is putting Sage-enhanced materials in the hands of teachers and students to assist them in teaching and learning mathematics. We will work with selected faculty at ten partner institutions by providing support for using Sage in their classrooms, offering technical support for Sage, assisting with the Sage-enhanced materials that we have created, and helping them write their own Sage-enhanced materials. Our teacher-authors and their students will receive access to Sage on a server dedicated to the UTMOST project, with dedicated technical support provided through this project.

We will recruit two groups of teacher-authors for UTMOST, with the first group beginning in Summer 2011 for a two-year commitment and the second group beginning in Summer 2012 for a one-year commitment, for a total of ten test sites. Teacher-authors will receive a stipend for their participation in UTMOST and support to attend workshops on using Sage in the classroom. In order to forge a strong relationship with the teacher-authors and provide the best possible support, each teacher-author will be paired with a member of the UTMOST team who will serve as a contact and adviser. This adviser will also make visits to the test site to provide support and gather evaluation data. We will expect the following from these participants.

- Teacher-authors will use Sage in their classrooms during their time with UTMOST. More specifically, they will use Sage-enhanced materials extensively in a semester-long course.
- Teacher-authors will write and test comprehensive Sage-enhanced curricular materials for the class that they are teaching.
- Teacher-authors will collect evaluation data from their students, and organize focus groups to gather feedback on the design and use of Sage-enhanced curricular materials. Focus groups will be led by the project member who is advising the teacher-author.
- Teacher-authors will offer training to others at their institution on how to use Sage and how to use Sage-enhanced materials in the classroom. Such training could be offered as departmental workshops or seminars.
- Teacher-authors from the first group will be selected to mentor the new teacher-authors in the second group.
- Teacher-authors will submit a report each year on their teaching experiences with Sage and new curricular materials.

5.3.1. *Recruitment plan.* Selection of the teacher-authors for UTMOST will be based on the following.

- A narrative statement by the applicant regarding rationale for applying, the course or courses in which they plan to use UTMOST materials, and their familiarity with Sage.
- Limited or no familiarity with Sage, since we aim to make it easy for all faculty to integrate mathematics software into their courses, not just those who already have this experience.
- A letter of support from the department chair indicating the flexibility to schedule faculty for the appropriate courses, a commitment to using Sage and new curricular materials in their department, and an awareness of evaluation activities.
- Consideration will be given to achieving a diverse mix of institutions and student populations, along with geographic proximity to a member of the UTMOST team.

5.3.2. *Initial test sites.* In response to a single posting in the sage-edu online forum, eight institutions expressed interest in being test sites. By casting a wider net, we are confident we could attract much more interest. From these eight institutions, we have selected and invited three to be part of our initial group of five sites. Letters of commitment from department chairs are included in the supplementary documentation. The three schools are:

- **Reed College** A highly selective, national liberal arts college where 50% of the students major in mathematics and science.
- **Steven F. Austin State University** A large comprehensive, regional institution primarily serving rural East Texas.
- **California State University, Dominguez Hills** A medium-sized public institution in the LA area that is "among the most ethnically-diverse universities in the US" [13].

We will have community colleges included in the remaining seven sites and have had discussions with one interested community college located very close to the University of Washington.

6. Evaluation

Formative and summative evaluation will be conducted at different phases of the project to address questions about the effectiveness of the project in creating and implementing the UTMOST model and Sage-enhanced materials for undergraduate mathematics teaching and learning. Dr. Judson will coordinate internal evaluation data-gathering, and will serve as the liaison to external evaluators from Ethnography & Evaluation Research at the University of Colorado at Boulder. The evaluation questions include:

(1) What aspects of the Sage-based tools and open materials are beneficial to instructors, what challenges do they face, and what kinds of support do they need in integrating these to mathematics teaching?
(2) How do instructors use and apply the Sage-integrated curriculum materials, and how do these benefit (or fail to benefit) their teaching of mathematics?
(3) How do the tools and materials impact instructors' content knowledge, pedagogical content knowledge, classroom instructional practice, and their students' learning?

Information will be gathered on both the processes and outcomes of the project at different stages, and results will be organized as case studies of participating instructors and their students. Formative components will focus on monitoring the quality of project activities, enabling the project to make mid-course corrections and plan for future development. Summative components will focus on the impact of the project on instructors' instruction and student learning at the test sites. Instructors and classrooms will be sampled, taking into account instructor interest and local institutional cooperation.

6.1. **Study design.** The study design includes pre- and post-surveys, follow-up surveys included in yearly self-reports, and interviews of the participating instructors. In addition, students will answer an online post-survey focusing on their experiences and gains in learning mathematics, including their classroom use of the Sage-enhanced materials. The design is informed by previous evaluation studies on professional development, education, and workshops in STEM fields [11, 24, 44, 73, 83] and on student outcomes of active instructional methods in undergraduate mathematics [30, 31].

6.1.1. *Pre-survey.* While registering for the summer workshops, participants will complete a short pre-survey to gather demographic and contact data, and information about their classes, institution, current teaching practices, and pedagogical needs. This information will also help the project leaders to plan workshops and later support.

6.1.2. *Post-survey and feedback.* Participants will complete a survey on the summer workshops so that project leaders can make adjustments for future workshops and implementation support. The survey will ask about participants' plans for using the model and Sage-enhanced materials to help guide later components of the study. Evaluators will observe the summer workshop and conduct a focus group with current and past participants.

6.1.3. *Follow-up survey and reports.* After using the model and Sage-enhanced materials for one year, the participants will file a report on their implementation, including some follow-up survey questions. They will report their use of the model and Sage-enhanced materials in their own classrooms and their future plans. Additional data on implementation will be gathered by the project team during site visits; they will conduct student focus groups using a protocol co-developed with the evaluators and share site visit notes that will be used as data sources.

6.1.4. *Student learning assessment.* Student learning will be evaluated after the first year of implementation with a post-survey based on the NSF-supported SALG instrument (DUE-0920801) [77], in which students report about their experiences of and learning gains from their mathematics course. This will provide information on student outcomes as well as formative feedback for the instructors using the model and Sage-enhanced materials in their classroom. A mathematics-specific version (SALG-M) has already been validated and used in a large evaluation study, and is sensitive to differences by student group and classroom practice [30, 40].

6.1.5. *Follow-up interviews.* Based on instructor and student responses on other measures, a sample of instructors will be interviewed to study factors that affect their success in implementing Sage-enhanced materials. The interviews will explore classroom use of the model and Sage-enhanced materials, impacts on instructional practices, and instructors' perception of students' response.

6.2. **Management and dissemination.** The evaluation will be led by Dr. Marja-Liisa Hassi from Ethnography & Evaluation Research (E&ER) at the University of Colorado at Boulder. E&ER has extensive experience evaluating large mathematics and science education projects (NSF DUE-0920126, DUE-0723600, DUE-0450088).

Drs. Hassi and Judson will collaborate with the project director, members of the project, and partner institutions to conduct the evaluation study. Dr. Judson and the project PIs will conduct site visits to partner institutions to observe and document the use of Sage-enhanced materials. Dr. Judson will have primary responsibility for extracting information from the site visits and workshop data to feed back to developers to improve the technology and classroom resources. Surveys and interview protocols will be prepared and conducted by Drs. Hassi and Judson. They will analyze all data and will prepare an annual report to document the evaluation activities and results. This will provide formative evaluation feedback to the project PIs and participating instructors to inform design decisions and mid-course corrections. A final report will gather results after the implementation of the tools and material by the second group of instructors in 2013. These findings will be shared also with the broader mathematics education community through a presentation and a coauthored article about the impacts of using CAS-integrated materials in teaching and learning undergraduate mathematics.

7. Dissemination

utmost is principally about dissemination—the focus issue is how we can enable the widespread implementation of approaches which have already been proven to work. In addition to working with our test sites, we will broadly disseminate the results of utmost through a variety of forums, several of which are already in place.

7.1. **Sage website.** The Sage website, `www.sagemath.org`, is an established venue for sharing materials related to Sage. This central location receives 90,000 visits each month, with 42% of the visits from the Americas and 45% of the visits from Europe. What we learn and create will be made available on the Sage website with open licenses. Furthermore, some of our work will be incorporated into Sage itself, which already has an effective distribution system. Complete, integrated documentation ships with every copy of Sage (such as the 4,701 page reference manual) and it is possible that much of our material, *including complete textbooks*, could ship as a core component of Sage itself. A library of high-quality interactive demonstrations will also be included in Sage, where they can be searched or browsed topically. These demonstrations will benefit from Sage's automated test suite that users routinely run on a wide variety of hardware.

7.2. **Sage Days workshops.** Sage Days, which are a series of conferences and workshops devoted to the development of Sage, are an ideal venue for dissemination. To date there have been nineteen Sage Days conferences, with nine more planned. These workshops now include sessions dedicated to using Sage in the undergraduate classroom, with the first such session taking place in December 2009 at the Clay Mathematics Institute during a week-long workshop on Sage and number theory. It attracted roughly thirty college faculty (and a few high school faculty) from around the Northeast, all eager to learn more about the use of Sage in educational settings.

Sage Days workshops devoted to training others to use our model and Sage-enhanced materials, and receiving and incorporating feedback and new ideas, will be entirely consistent with the way Sage Days have been used to drive Sage development. The first Sage Days workshop sponsored by the project will be held in Summer 2011 and will primarily involve UTMOST and representatives from the first five test sites. A second Sage Days event will occur in Summer 2012, involving UTMOST and all ten test sites, as an opportunity to discuss the project, initiate the second group of five test sites, and begin the mentoring relationships between the sites in the two groups. Faculty, students, and Sage developers with interests in education will be encouraged to attend these events.

7.3. **National workshops.** We will apply to offer special sessions and workshops on the use of Sage in the classroom at the national Joint Mathematics Meetings and MathFest conferences, where we will be able to share the results of UTMOST with the undergraduate teaching community. We will also publish articles about the impact of using our Sage-enhanced materials on the teaching and learning of undergraduate mathematics. Drs. Beezer and Grout are part of a team delivering a workshop in Summer 2010 on how to use Sage in the classroom as part of the NSF-funded Mathematical Association of America Professional Enhancement Program (DUE-0817071). We will apply to lead future PREP workshops that will incorporate the results of UTMOST. Notably, Stein has been invited to propose an AMS Short Course on Sage for the 2011 Joint Meetings.

7.4. **AIM open textbook initiative.** The American Institute of Mathematics (AIM) is a respected NSF-funded institute that has supported both leading research mathematicians and mathematics educators at all levels. As the sponsoring organization for UTMOST, AIM will conduct a pilot project to test the feasibility of sponsoring and supporting a series of open mathematics textbooks. A hindrance to the adoption of open textbooks is the lack of an acknowledged authority to vouch for the content. Usually this is a role played by a publisher, so a recognized organization within the mathematics community with a trusted reputation, such as AIM, will be another component to broadly disseminating our materials. This has the potential to positively impact open textbooks, in mathematics and more generally.

AIM will constitute an editorial board to guide this initiative by developing policies and programs to promote the development of high quality textbooks and associated course materials for mathematics courses at all levels of the post-secondary curriculum. During the first year, the new board will oversee the creation of a website for publicizing the project and for electronic dissemination of the materials. In order to compete with the traditionally published texts, we expect to establish a way to provide examination copies in hard copy and to provide course outlines, syllabi, exercise sets, and exams from instructors who have already used the books. We anticipate that the board should be able to review and edit approximately five textbooks in the first year.

Because the faculty in community colleges are much more sensitive to the high cost of traditional texts and more likely to adopt open source alternatives, we expect the editorial board to begin to work with the Community College Consortium for Open Educational Resources (CCCOER), which is an initiative of the Foothill-De Anza Community College District, where AIM is also located. This consortium recently received a two-year grant of $1.5 million from the William and Flora Hewlett Foundation to manage the Community College Open Textbook Collaborative.

During the second and third years, we plan to increase the number of books included in the series and to solicit and develop textbooks for at least one high-enrollment mainstream course. The board will also consider the logistics of distributing texts to students. Although many may be satisfied with electronic versions to read on their computer screen, others will want traditionally bound texts, and the board will develop means to provide them, perhaps in partnerships with custom publishing firms, such as Lightning Source [45]. Determining best practices for making open textbook metadata available for the National Science Digital Library (NSDL) is another aspect of dissemination to explore.

8. Sage infrastructure

The Sage notebook interface and server configurations are key elements of a successful experience for students and faculty using Sage in their courses. A portion of our work will be to improve Sage itself in those areas where the improvements *directly support* educational applications of Sage. Discussions and initial queries in the sage-edu email list indicate that faculty around the world are excited about setting up Sage servers for their students' use [70]. We see three main areas for work on Sage.

8.1. **The Sage library.** Experience has shown us that teaching with Sage invariably suggests new capabilities or exposes needed commands. Our work enhancing textbooks, and our teacher-authors at test sites, will certainly suggest extensions and corrections to the Sage library. We will continue to refine and enhance Sage at an accelerated pace with support from this grant. All changes will undergo the existing code review process and will then become permanent contributions to Sage, maintained and further extended by the worldwide Sage community. We have already identified specific improvements for linear algebra, abstract algebra, calculus, differential equations, number theory, and combinatorics, as well as broader areas like 2-D and 3-D graphics. Targeted fixes and improvements will greatly improve the educational experience for students. In many cases, undergraduate students can, and already have, contributed code and other suggestions; we plan to employ several students in these efforts with funding from this grant.

8.2. **Notebook development.** The Sage notebook interface is a powerful tool for experimentation and collaboration. Stein worked full-time during Fall 2009, supported by the University of Washington, on improving the robustness and scalability of the notebook. In Fall 2009, we have typically seen two thousand new accounts created on `www.sagenb.org` every month. The notebook interface is the face of Sage for students and we intend to improve it by fully implementing labels and folders, support for much more sophisticated interactive demonstrations, offline access to worksheets, user-customizable styles, and enhanced security and authentication support.

8.3. **Sage servers the easy way.** Sage servers allow a user with just a standard web browser to use Sage over the network. Initiating and maintaining a server will be made as simple as possible, making it easy for faculty and system administrators to get their students started with Sage.

A virtual computer solution provides a very easy way to set up and securely maintain a Sage server. VirtualBox [80] is a leading free, open-source product for creating and running virtual computers. We will create a VirtualBox Sage notebook server appliance with a graphical interface. Users will be able to easily install this appliance on Windows, Linux, Mac OS X, and Solaris (x86) servers. The management interface will provide clear and easy documentation about setting up this server, creating new notebook servers for specific classes or instructors, starting and stopping notebook servers, monitoring resource usage, adding users and authentication frameworks (such as LDAP), and upgrading Sage with minimal user downtime.

For users at our ten test sites, we will create a new mailing list for server administration issues, and in addition to the help we provide ourselves, we will hire a knowledgeable system administrator to answer support questions on a daily basis.

Several specific goals for notebook and server development include the following.

- We will improve the notebook so that it will robustly handle up to 250 simultaneous users viewing worksheets and doing typical computations for an undergraduate course when running on a single high-end server, as demonstrated by a robust automated test suite.
- We will implement management tools so administrators can manage the notebook load and better balance resources.
- We will create tools for educators to get automatic feedback on their students' use of Sage.

In summary, we propose to provide high-quality software infrastructure and support to educators.

9. Qualifications and Preparation

The UTMOST team consists of five mathematics faculty and an experienced evaluator of STEM education initiatives. The five faculty together have many years of experience teaching undergraduates at a wide range of institutions, four are active Sage developers (including its founder), three are authors of open textbooks, all have significant mathematics research experience, and one specializes in mathematics education research. As Sage developers, they are in a position to quickly and accurately shape changes in Sage based on the experience of working with faculty at other institutions that are new to Sage. Each is familiar with, and has extensive experience using, open source software and tools in their teaching and other professional activities. Besides the applicability of this experience to the project's central theme, they are also adept at working collaboratively while still being separated geographically. Working together, they have the wide range of complementary experience and skills that will make UTMOST a success.

Dr. Robert Beezer, Professor of Mathematics at the University of Puget Sound, is an undergraduate teacher with 31 years of experience, an active researcher in algebraic graph theory, one of the first open textbook authors, and a Sage developer. He began writing his open source linear algebra textbook in 2004 and has assisted Dr. Judson with the recent release of his very successful open source abstract algebra text. He began using Sage in 2007 and began contributing code in early 2009. He will lead the technical process of converting textbooks from LaTeX to Sage worksheets, producing a simple system for other authors to use. The pedagogical implications of this new capability will be explored as he incorporates Sage into existing textbooks on topics he teaches frequently, such as linear algebra, abstract algebra, combinatorics, calculus and cryptography. He will continue to contribute code to Sage where the new functionality enables a more complete experience for undergraduate students, and will suggest, review and test the project's improvements to interactive demonstrations and the notebook interface.

Dr. Thomas Judson, Associate Professor of Mathematics at Stephen F. Austin State University, is an active researcher in both mathematics and mathematics education, with 32 years of teaching experience. He is the author of an open source undergraduate abstract algebra textbook, and will work with Dr. Beezer to produce Sage-enhanced materials for abstract algebra. Dr. Judson has worked extensively with undergraduate mathematics teachers and has mentored graduate students in the teaching of mathematics. In addition to his teaching and research accomplishments, Dr. Judson brings added experience in working with diverse groups both in the US and abroad. He will work with Dr. Hassi to guide research and evaluation efforts for UTMOST. With experience in working with undergraduate faculty and graduate student mentoring, Dr. Judson will also provide insight and assistance in the implementation of Sage-enhanced materials in the undergraduate classroom. He will direct project efforts connected to research in mathematics education.

Dr. William A. Stein, Associate Professor of Mathematics at the University of Washington, is a leader in both computational and theoretical number theory, and the author of two published number theory books. He started the Sage project in 2005, and has passionately led its development ever since, including co-organizing over 25 workshops during the last 4 years and directing dozens of Sage development projects by students at Harvard, UC San Diego, and U of Washington. Dr. Stein will implement improvements to Sage, direct work by students, contribute curricular materials he has authored, supervise test sites, and co-organize workshops.

Dr. Jason Grout, Assistant Professor of Mathematics at Drake University, is an undergraduate teacher, an active researcher in combinatorial matrix theory and graph theory, and a Sage developer. Dr. Grout has contributed extensively to the Sage code base over the last three years in the linear algebra, graph theory, graphics, and notebook interface components of Sage. Dr. Grout has used Sage in research and in teaching a number of undergraduate courses. He has also given numerous presentations and tutorials on Sage, and in Summer 2010 will be co-directing an MAA PREP workshop with Drs. Beezer and Karl-Dieter Crisman to help participants develop new curricular materials in Sage. Dr. Grout will implement improvements to Sage, direct work by students, supervise test sites, help organize workshops, and contribute curricular materials.

Dr. Marja-Liisa Hassi, Ethnography & Evaluation Research, University of Colorado at Boulder, has a Master's Degree in Mathematics and a PhD in Education. She has expertise in both theory and methodology of mathematics education research, as well as teaching experience in education, mathematics education, and research methods for undergraduate and graduate students. Her recent publications address theory and measurement of affect in undergraduate and adult mathematics learners and comparative student outcomes of inquiry-based and traditional undergraduate mathematics courses. She will be assisted by Dr. Sandra Laursen, co-director of E&ER and an experienced evaluator of STEM education initiatives in higher education.

Dr. Kiran S. Kedlaya, Associate Professor of Mathematics at Massachusetts Institute of Technology and UC San Diego, is a leading researcher in arithmetic algebraic geometry. He has received an NSF CAREER grant, a PECASE award, and a Sloan Fellowship, and was selected to speak at the 2010 International Congress of Mathematicians. Dr. Kedlaya brings extensive experience in undergraduate and graduate education at all levels, from calculus to mentoring Ph.D. students. He is also deeply involved with mathematics competitions and other programs for exceptional students, such as the USA Math Olympiad, the Putnam competition, and the Art of Problem Solving Foundation. In addition, Dr. Kedlaya brings extensive knowledge of the Sage system, having been an active user and developer since 2005. Dr. Kedlaya will review improvements to Sage related to the project, supervise test sites, and provide additional overall guidance.

9.1. **Grant experience and support.** The UTMOST project members have a wide variety of experience in administering grants.

- As a sponsoring organization, the American Institute of Mathematics has a ten-year history of successfully administering, supporting and executing 38 NSF grants in mathematics.
- Dr. Stein has successfully administered many grants supporting Sage development from varied organizations such as NSF, UC San Diego, University of Washington, Google, Sun, Microsoft, and the US Department of Defense. National Science Foundation grants include awards from the SCREMS program for the `www.sagenb.org` computing cluster (DMS-0821725), the FRG program (sponsored by the American Institute of Mathematics, DMS-0757627) and the COMPMATH program to fund two postdoctoral positions (DMS-0713225).
- Dr. Judson works with middle and high school mathematics teachers from high-needs school districts to help them become teacher-leaders in their schools and districts, and prepare them to deliver pedagogical content and mathematical content to their colleagues. He also collaborates with the PIs on two NSF grants to help direct the mathematics education research component (DUE-0934878, DUE-0227128).
- Dr. Beezer has been awarded a competitive year-long sabbatical leave from the University of Puget Sound for the 2010-11 academic year. This will allow him to begin converting textbooks, increasing the possible textbooks available for test sites to use beginning in the Fall 2011 term.
- Dr. Grout co-directed the 2008 IMA Participating Institution Summer Program for Graduate Students: Linear Algebra and Applications at ISU (program was supported by IMA and NSF; Dr. Grout was supported by an NSF conference grant).
- Drs. Beezer and Grout will co-direct (with Dr. Karl-Dieter Crisman) an MAA PREP workshop on Sage in Summer 2010, as part of a program funded by the NSF (DUE-0817071).
- Dr. Hassi is an expert in the research and evaluation of mathematics education and is currently working with a large evaluation study focused on inquiry-based learning and teaching of undergraduate mathematics at four large research universities [31, 39]. She is also currently working with an evaluation study of NSF DUE-funded workshops on inquiry-based learning for instructors (DUE-0920126).

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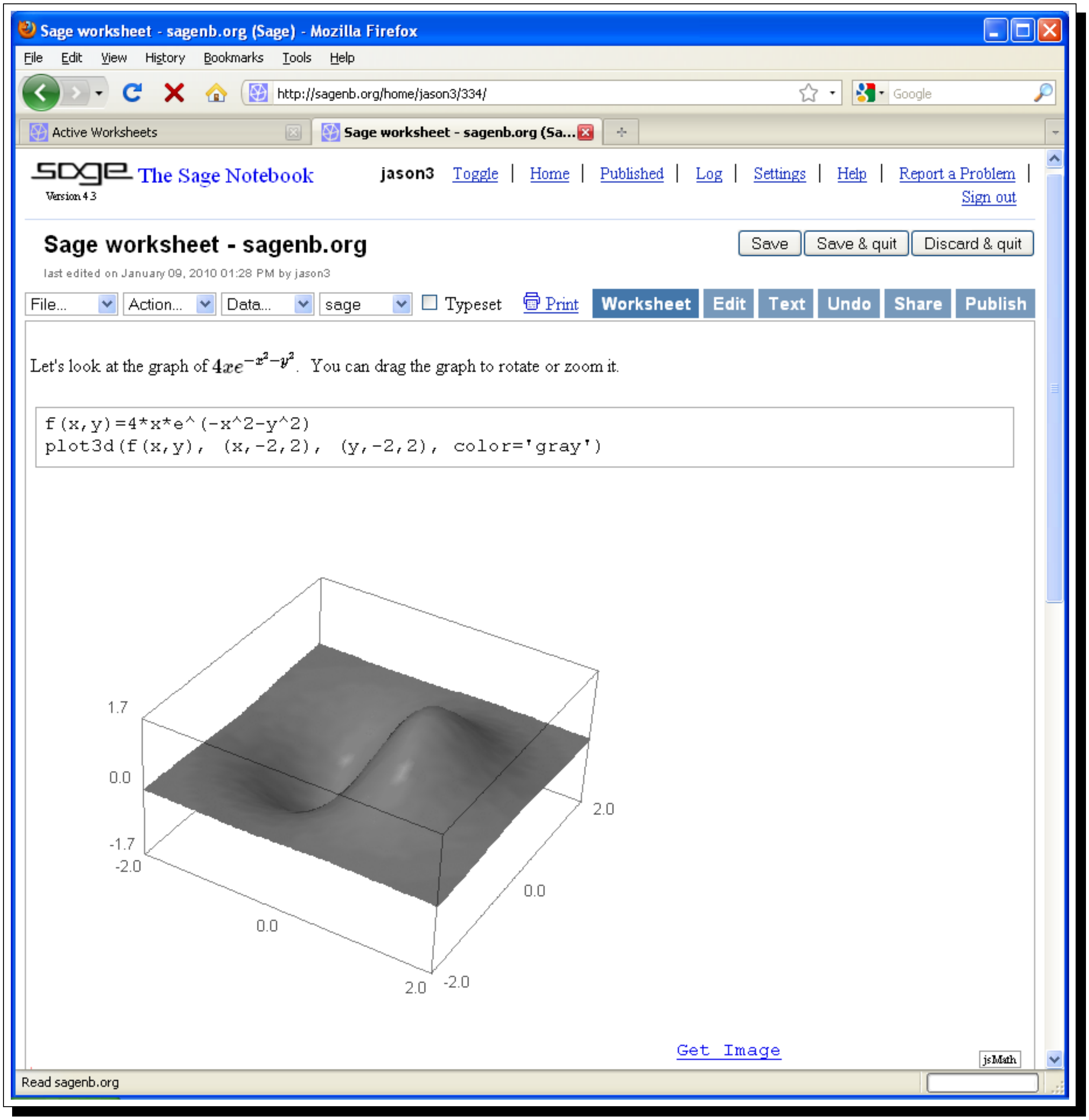
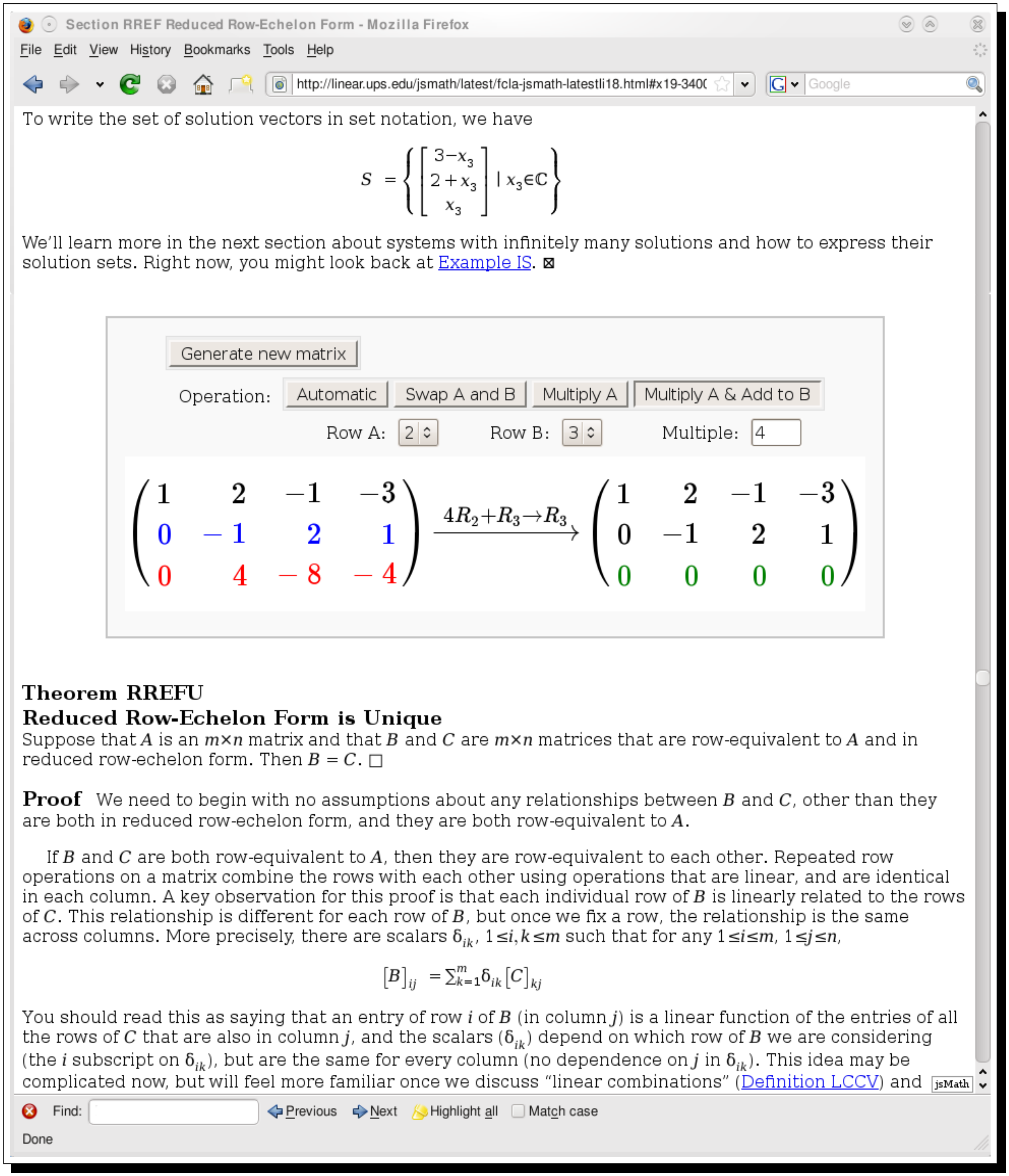
UTMOST Supplementary Documentation: Sage Notebook Screenshot



Screenshot of the current Sage notebook running in Firefox on Windows

UTMOST Supplementary Documentation: Sage Textbook Screenshot



Screenshot of a Sage-enhanced mathematics open textbook prototype